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ON THE MOTION OF THE KOVALEVSKAIA GYROSCOPE IN A PARTICULAR DEGENERATION CASE*

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Motion of the Kovalevskaia fast top was considered in /1/ in the case of $e_3 = e_4 = s_2$. Below, we investigate a similar motion under the condition

$$e_3 = e_5 = s_2 \quad (e_1 \neq e_2) \tag{1}$$

where $e_i \, (i = 1, \ldots, 5)$ are the roots of polynomial $\Phi(s)$ /1/, and s_1, s_2 are the Kovalevskaia variables. The quantity s_1 can be determined using the equation

$$\frac{ds_1}{\sqrt{\Phi_1(s_1)}} = \frac{i}{2}\, dt, \quad \Phi_1(s_1) = 8(s_1 - e_1)(s_1 - e_2)(s_1 - e_4) \tag{2}$$

Let us impart to the body a high initial angular velocity ω. The initial conditions then assume the form

$$p_0 = \omega a, \quad q_0 = \omega b, \quad r_0 = \omega c, \quad \gamma_0, \ \gamma_0', \ \gamma_0''$$

where a, b, c are the directional cosines of the initial axis of rotation.

With formulas (1) taken into account the initial conditions must satisfy the following four equations:

$$a^2 + b^2 + c^2 = 1, \ \gamma_0^2 + \gamma_0'^2 + \gamma_0''^2 = 1, \ 3l_1 - 2l^2 = -k, \ c - 2l\gamma_0'' = 0 \tag{3}$$

Constants l_1, l, and k are defined by the well-known formulas in /1/. The third equation of system (3) represents the relation for which $e_3 = e_5$, and the fourth is determined by the integral $r - 2l\gamma'' = 0$ /3/, which occurs under condition (1) when $q \neq 0$ (if $q = 0$ we have the Bobylev–Steklov case).

We obtain

$$a = -\frac{\mu}{\sqrt{4 - 3c^2}} + \mu^2(\ldots), \quad b = \sqrt{1 - c^2} + \mu^2(\ldots), \quad \gamma_0 = 0$$

$$\gamma_0' = \frac{2\sqrt{1 - c^2}}{\sqrt{4 - 3c^2}} + \mu^2(\ldots), \quad \gamma_0'' = \frac{c}{\sqrt{4 - 3c^2}} + \mu^2(\ldots)$$

where $\mu \sim \omega^{-2}$ is a small parameter and c is an arbitrary positive parameter, and $c^* < c \leqslant 1$ ($c^* = 2\mu^{1/2} + \mu^{3/2}(\ldots)$) is determined from the condition that $e_1 = e_2$. Note that for this interval of variation of c and the value s_1 is a periodic function of time.

From Ketter's equations /2/, equation (2), and the equations of motion of the body and of their first integrals /1/ we can obtain final expressions for the variables of problems and their expansions in series in powers of the small parameter μ.

Let us now use Euler's angles θ, φ, Ψ for defining motions of the solid body. We have

$$\theta = \theta_0 - \frac{2\mu}{c} \sin \frac{\omega c}{2} t + \mu^2(\ldots), \ \cos\theta_0 = \frac{c}{\sqrt{4 - 3c^2}}, \ \Psi = \Psi_0 + \frac{\sqrt{4 - 3c^2}}{2} \omega t \tag{4}$$

$$\varphi = \frac{\omega c}{2} t - \omega\mu \frac{4\sqrt{1 - c^2}}{c^2\sqrt{4 - 3c^2}} \cos \frac{\omega c}{2} t + \omega\mu^2(\ldots) \ (\varphi_0 = 0)$$

To obtain a geometric interpretation of the obtained solution we draw on a unit radius sphere two parallels at angle $\pm 2\mu/c$ from the median parallel θ_0. The trajectory of the body z axis can be then expressed in the form of the sine curve

$$\theta - \theta_0 = -2\mu c^{-1} \sin[\cos\theta_0 (\Psi - \Psi_0)] \tag{5}$$

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of period $T = 2\pi/\cos\theta_0$ and alternately touching the two extreme parallels. As implied by formula (4), the proper rotation of the body does not greatly differ from uniform rotation at the high angular velocity $\omega c/2$. It follows from Eq.(5) that $\theta_0 = \pi/2 - \mu^{1/2} + \mu^{3/2}(\ldots)$ as $c \to c^*$, with the band width equal $4\mu/c$ increasing to $2\mu^{1/2}$ and period T increasing and approaching $2\pi\mu^{-1/2}$. When $c = 1$ we have the Delone case /4/.

Let us compare the motion obtained (for $e_3 = e_5 = s_2$) with the motion in the case of $e_3 = e_4 = s_2$ considered in /1/. In the latter case the trajectory of the z axis on a fixed sphere of unit radius can be represented in the first approximation by the curve $\Psi = -{}^1/_2\mu a\theta$. The point moves on that curve as on a meridian, at high angular velocity ω. The meridian plane slowly rotates clockwise about the vertical at angular velocity $\mu a\omega/2$. The proper rotation of the body is of the form of oscillations with low amplitude rotation at high frequency.

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REFERENCES

1. ARKHANGEL'SKAIA M.Iu., On the motion of the Kovalevskaia top in a particular degenerate case. Vestn. Mosk. Univ., Matem. Mekhan., No.5, 1981.
2. GOLUBEV V.V., Lectures on the integration of equations of motion of a heavy solid body about a fixed point. Moscow, GOSTEKHIZDAT, 1953.
3. APPEL'ROT G.G., Not entirely symmetric heavy gyroscopes. In: Motion of a Solid Body About a Fixed Point. Moscow—Leningrad, Izd. Akad. Nauk SSSR, 1940.
4. ARKHANGEL'SKII Iu.A., On the motion of Kovalevskaia's gyroscope in the Delone case. PMM, Vol.36, No.1, 1972.

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